ISM2011

Rapid Thermalization by Baryon Injection in Gauge/Gravity Duality

Norihiro Iizuka
(*CERN*)

Based mainly on **arXiv:1012.4463** with
Koji Hashimoto and ***Takashi Oka***
commenting also on
arXiv:1003.4988, 1005.4412, 1006.3612 with
Koji Hashimoto and ***Piljin Yi***

Introduction & Motivation

- Understanding non-equilibrium statistical mech. is one of the most difficult problems
- Why difficult?
- It involves many bodies, it involves strong interactions where perturbation breaks down and it involves non-linear time-evolution
- These are quite difficult settings where we usually do not have clue to simplify things

⇨ You may get help from (thought) experiments

Data help?

- One of the recent interesting experiments for string theorists are *RHIC* experiments where quite rich QCD physics were found, where many unexpected/surprising aspects of QCD are revealed (ex. Jet quenching, energy loss)
- They form strongly interacting quark-gluon plasma (QGP) which behaves as `fluid', rather `gas'.

→ Rather strongly interacting

Data help?

- This implies that QGP is rather strongly interacting
- Holography or AdS/CFT might works? even though we do not have realistic QCD dual..?
- It looks that AdS/CFT works at least qualitatively to understand some of the nature of this experiments more than you thought… (ex. Jet quenching energy loss of light quarks)

→ *Exciting to connect strings to real world*

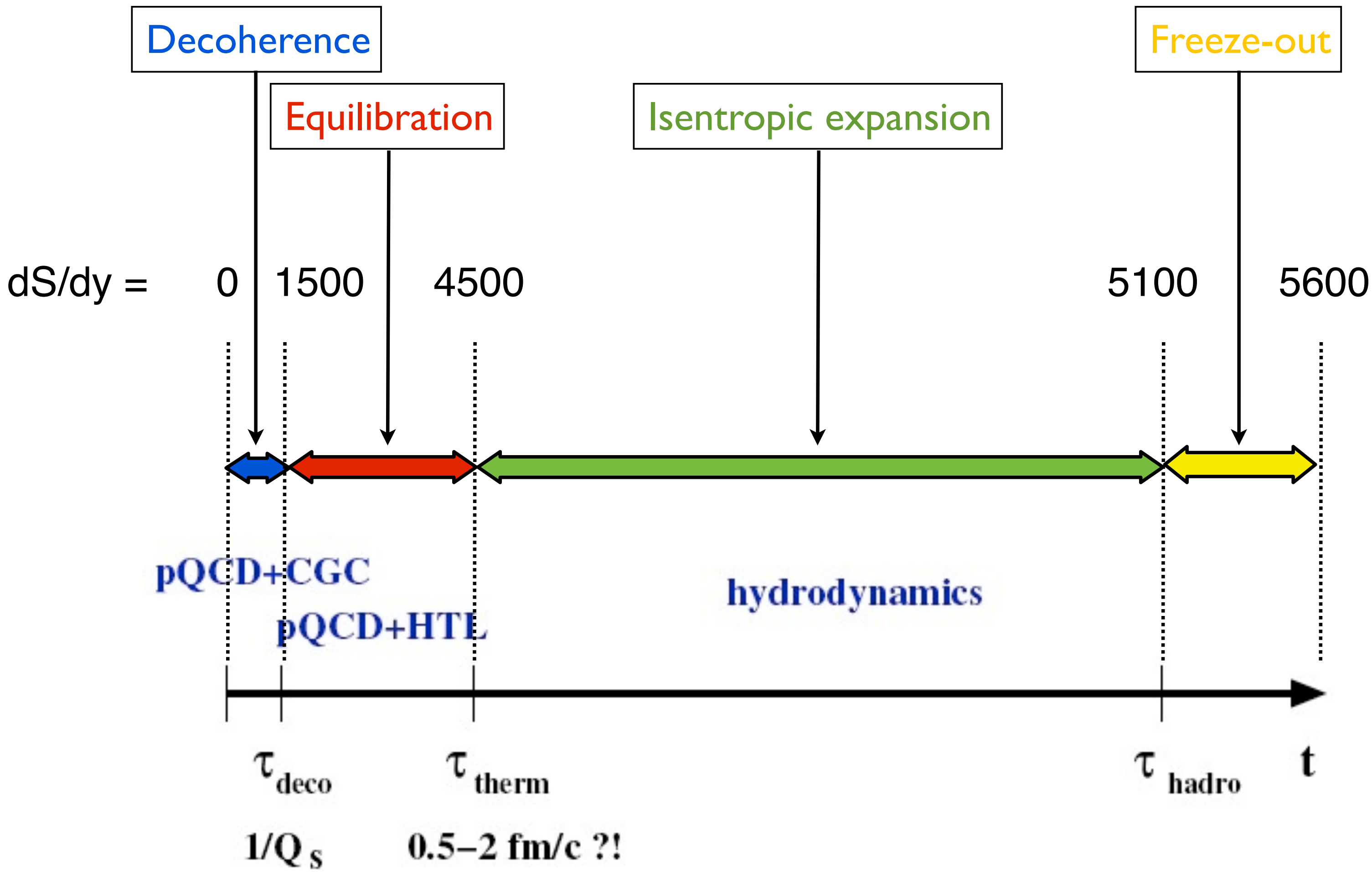
Data help?

- The key is that for some nature of the QGP plasma (=deconfined plasma), qualitative physics can be learned even from CFT lacking confined phase
- Even conform CFT (= AdS_5) may be useful to learn about something about real world QCD!

On the other hand,

- In RHIC data, what is the most challenging and surprising results is that QGP forms in a very very fast stage ~ 2 fm/c ~ 10^{-23} [s] after the heavy ion (Au) collisions

Thermal history of RHIC



The challenge

- Many of the RHIC physics is based on the assumption that if we assume that QGP thermalization occurs in very early stage as ~ 2 fm/c, then we can explain the evolutions of experimental data like observed final elliptic flow universally by hydrodynamic models

Goal of this talk

- In this talk, we use AdS/CFT and calculate strongly coupled gauge theory time-scale for the thermalization (QGP formation) for `mesons'
- QGP thermalization means that system loses all information of initial states
- Due to the holographic correspondence, this is equivalent to the black hole horizon formation in the bulk AdS side

In summary;

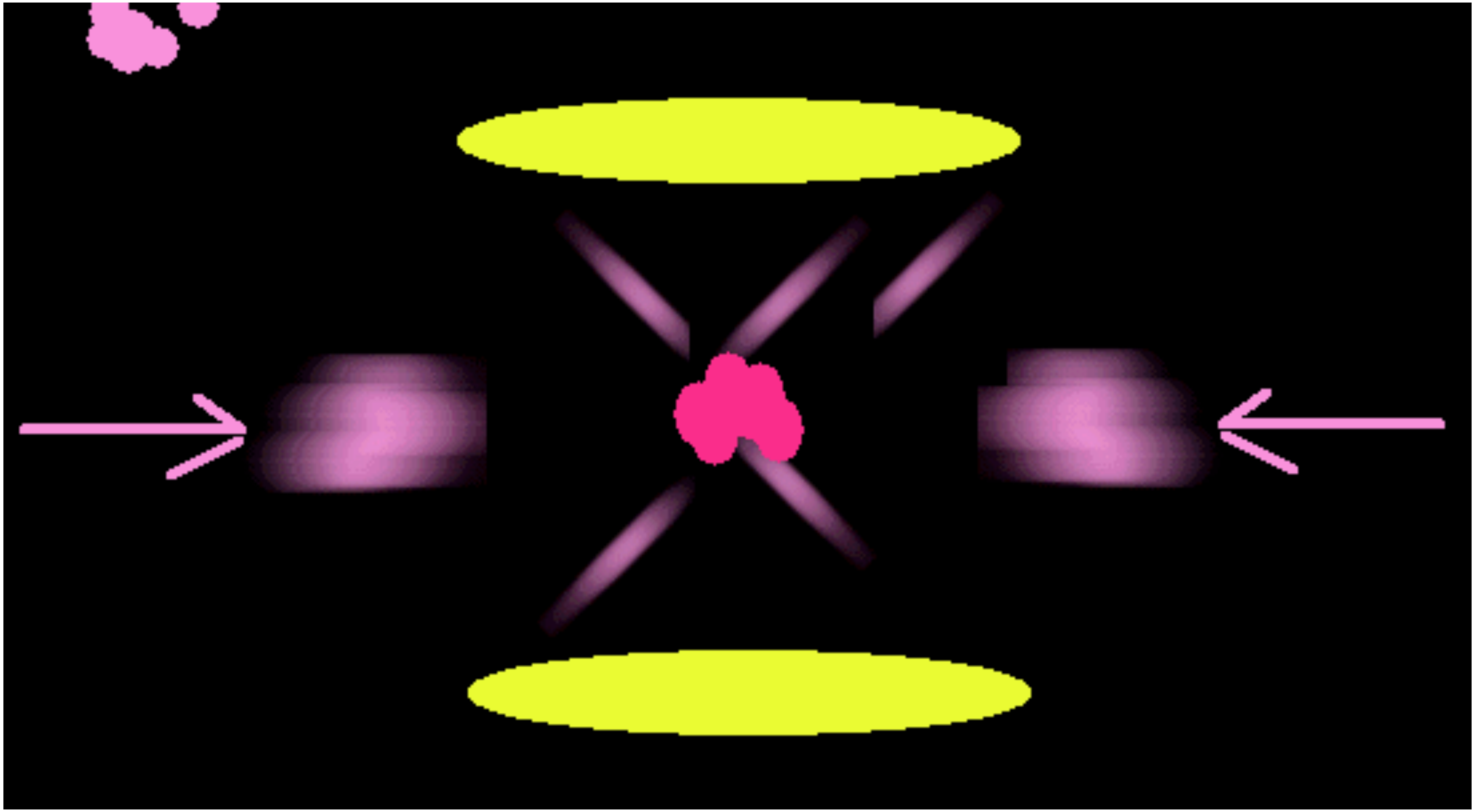
The thermalization time-scale is given by hand to fit to the data, due to the difficulty of nonequilibrium strong coupled QCD

Our strategy

Holographic QCD approach;

We approximate RHIC collisions as ``local homogeneous baryon-number chemical potential sudden change”

Heavy ion collision



The key points;

- We consider thermalization for mesons due to baryon number (~ quark number) chemical potential sudden change (i.e., quantum quench)

  (c.f. Sumit's talk, Das, Nishioka, Takayanagi)

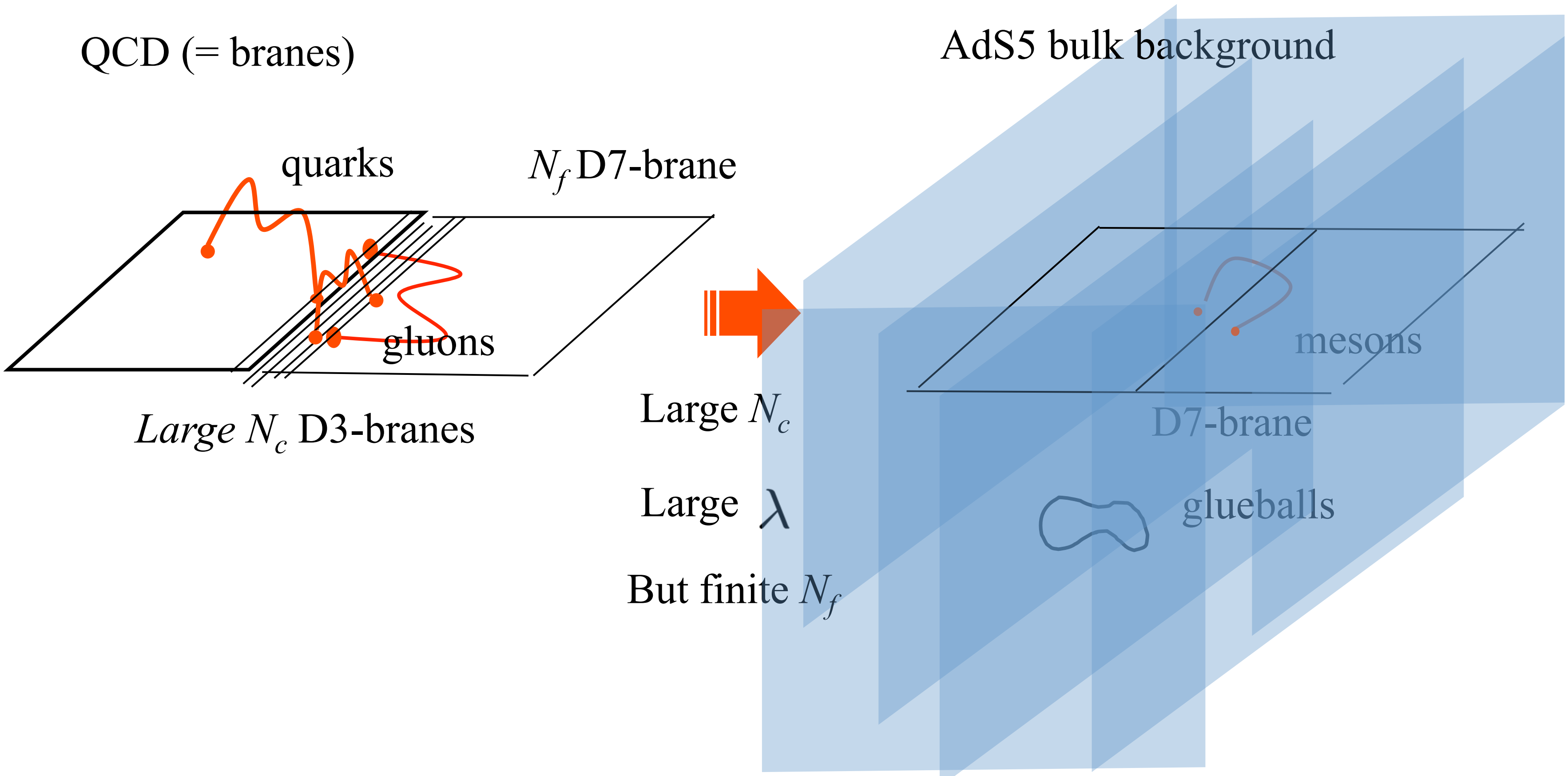
- We take large color N_c but keep flavor N_f finite
- We consider thermalization (horizon formation) on probe flavor branes, namely, this is the horizon for meson d.o.f. (on probe brane)

The key points;

- Remember that mesons live on the N_f flavor brane in the warped AdS
- Therefore we consider the dynamics only on the flavor brane
- Since N_f is kept fixed, back-reaction bulk graviton is totally negligible!
- This is much easier than bulk black hole formation!

Toy Model of holographic QCD

- Mesons and quarks are open strings



Toy Model of holographic QCD

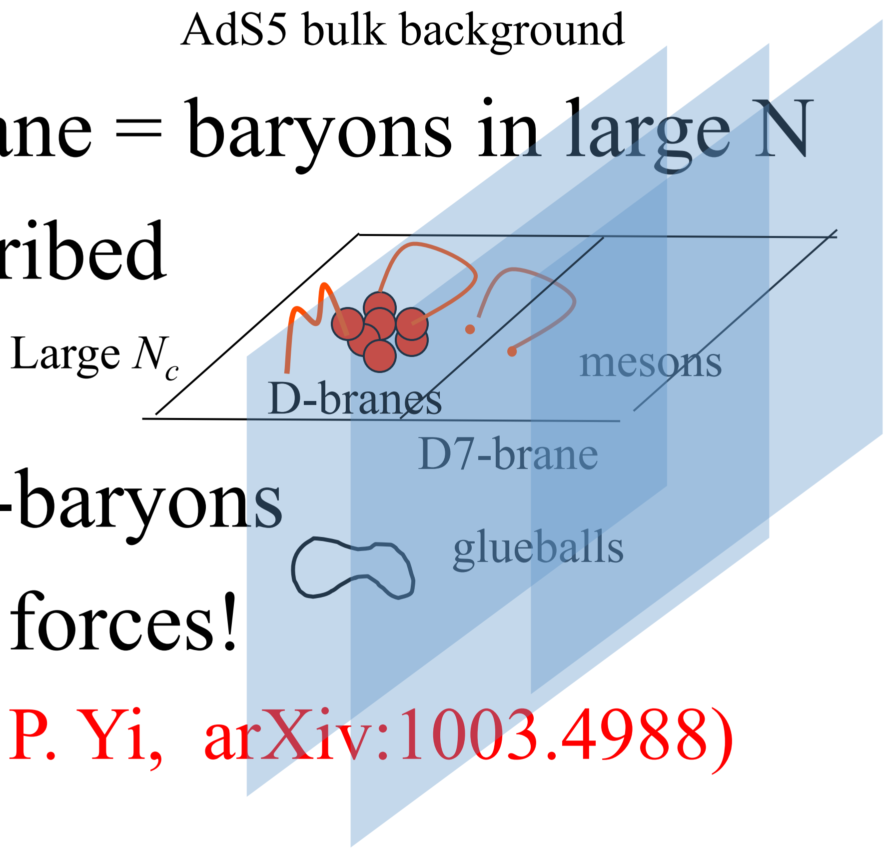
- D-branes wrapping on sphere in AdS times S are baryons (Witten)
- D-branes on flavor brane = baryons in large N
- *k* D-brane can be described

*k*x*k* matrix Q.M.

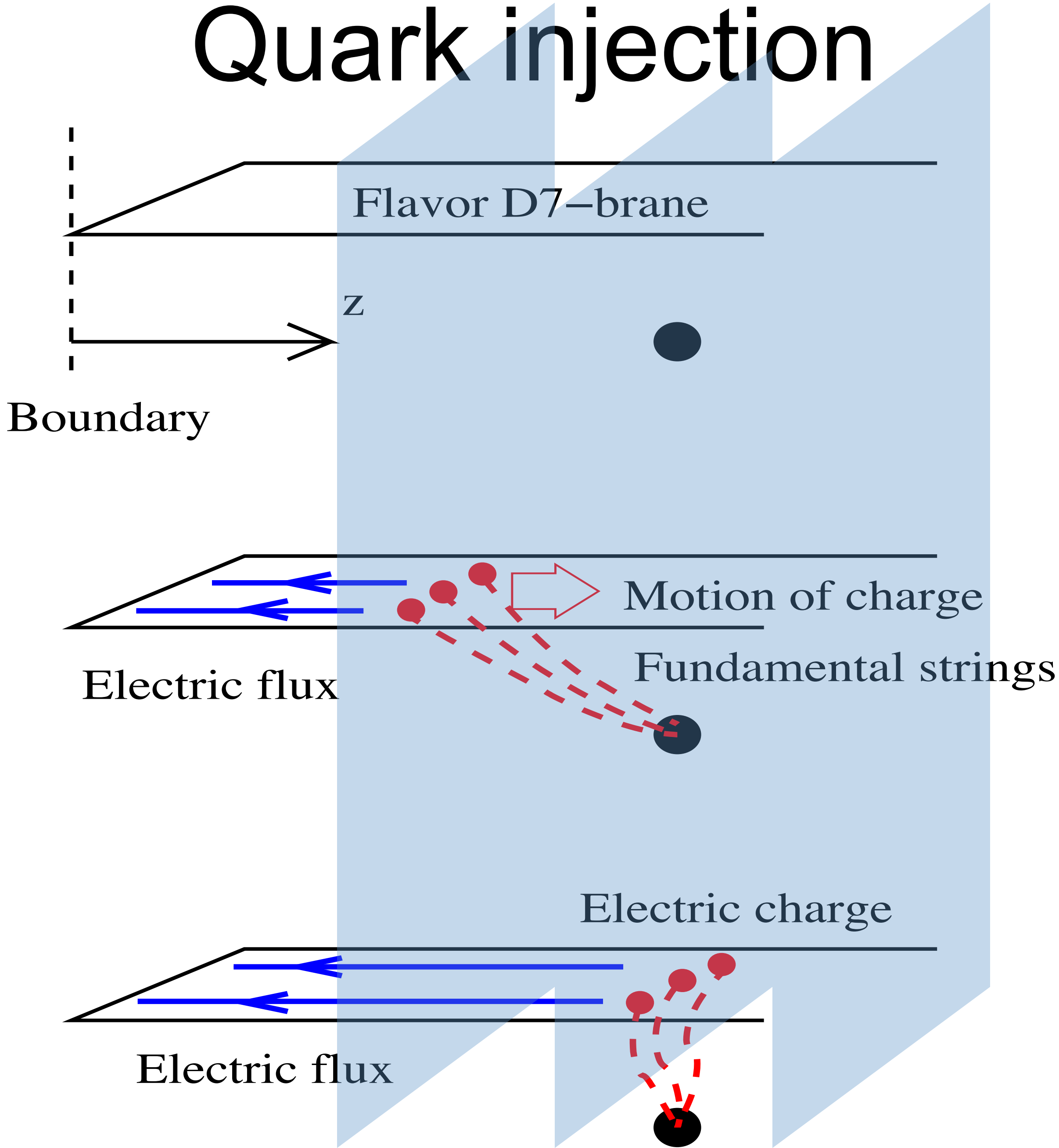
➡ Matrix Q.M. for multi-baryons

which describes nuclear forces!

(work with K. Hashimoto, P. Yi, arXiv:1003.4988)

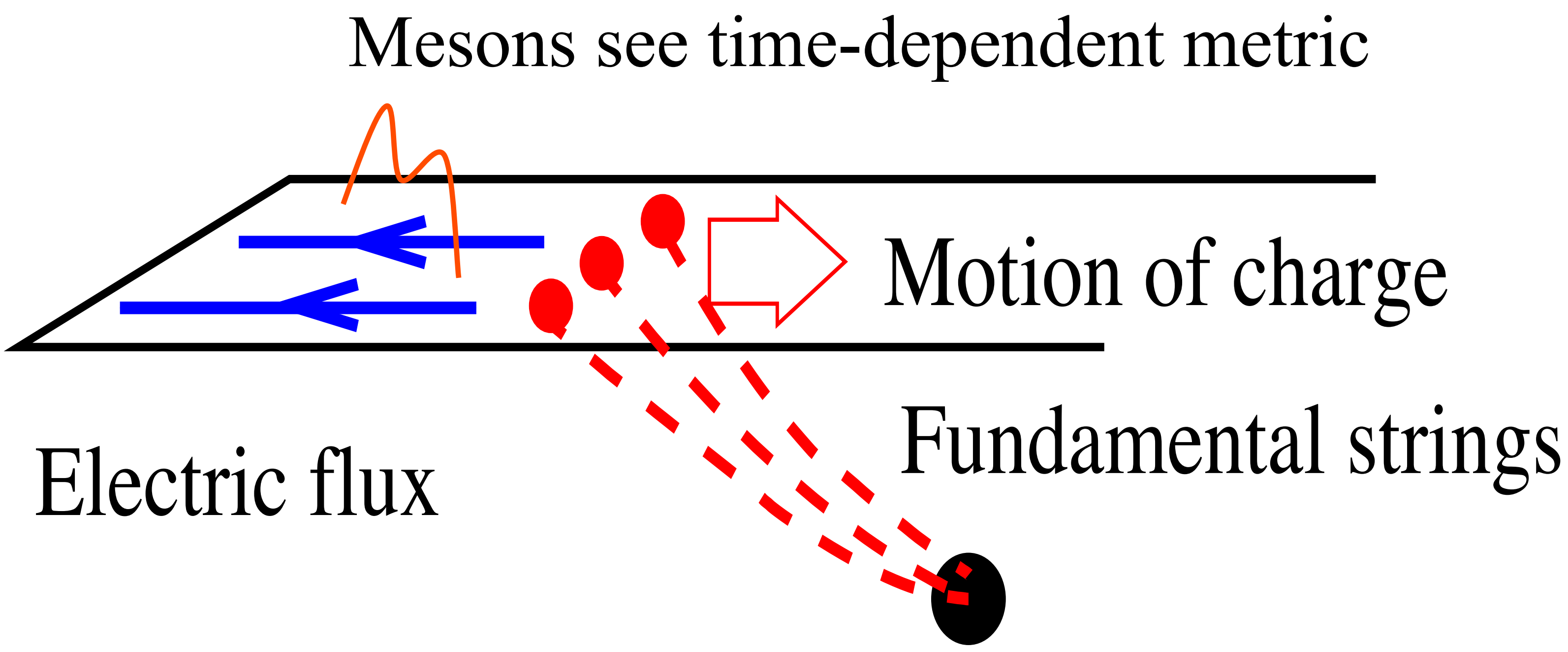


Quark injection



Mesons effective metric

Boundary



The key points;

- Baryon number chemical potential injection is done by F-string injection which is quarks
- F-string induces flux
- In AdS/CFT, boundary global symmetry corresponds to local symmetry in bulk

The key points;

- Baryon number charge in boundary theory corresponds to local gauge charges on the bulk sourced by F-strings
- Injection of F-strings in bulk induces time-dependent flux on bulk flavor branes

The key points;

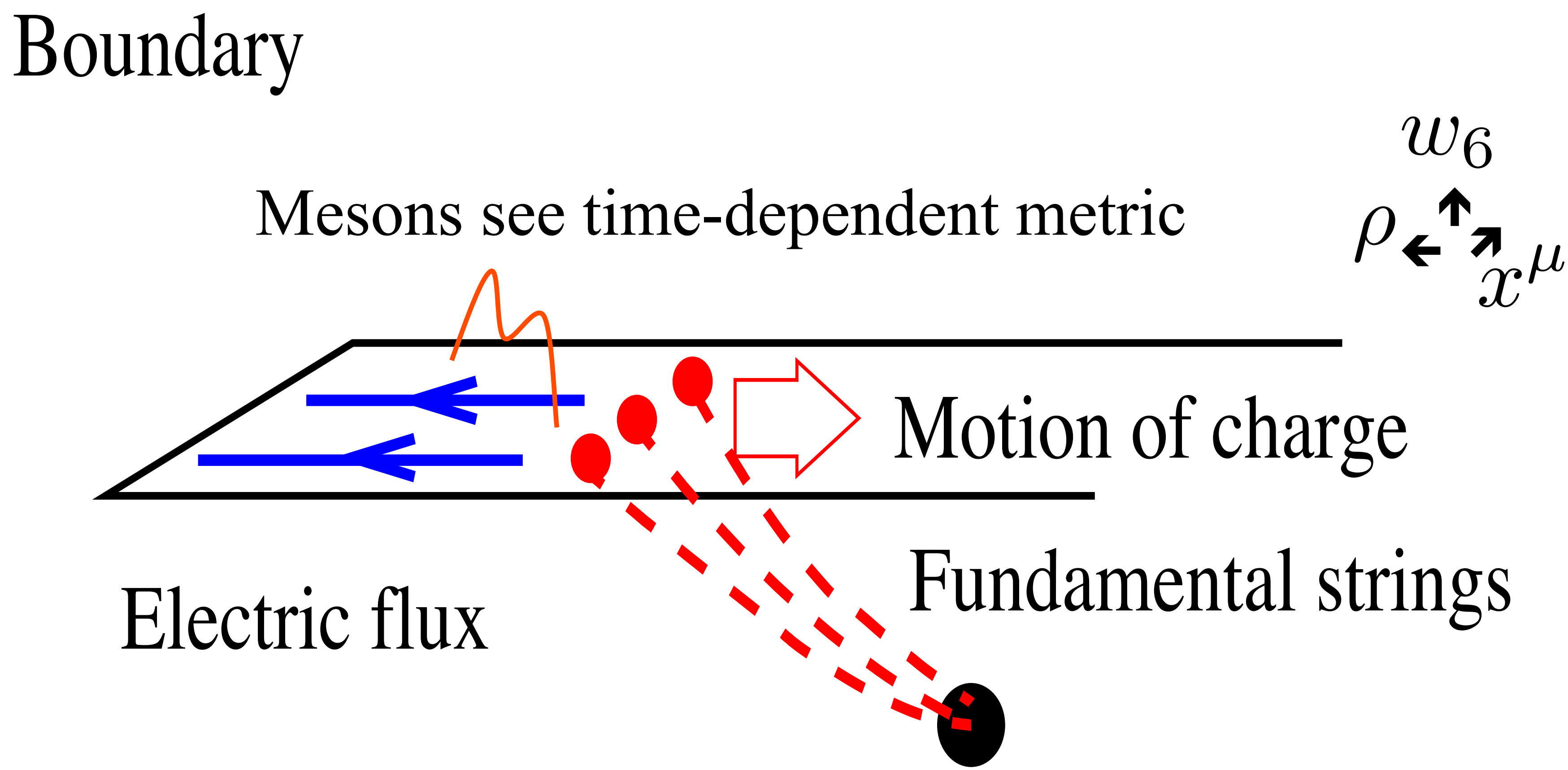
- Through the DBI action, small fluctuation on the flavor branes (which is mesons) ``feel'' this nontrivial flux and therefore their effective metric is also time-dependent
- We can calculate if this effective metric shows apparent horizon or not, and *if horizon are formed and does not disappear in finite time, we can regard this as indication of thermalization of mesons*

Our strategy

- We approximate the heavy ion collision as locally homogeneous baryon number chemical potential sudden change
- For that, we inject infinitely heavy quarks into the bulk by hand (<= our input)
- This induces the time-dependent flux (of baryon charge) on the flavor brane
- This effectively modify the metric for the mesons, which sees horizon

Plan of the talk

- Introduction & Main punch lines
- A bit technical things; Quark injection & Horizon formation on the flavor brane & Thermalization time-scale order estimation
- Heavy ion collision example & Comparison with Data
- Discussion on Universality



$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}(d\rho^2+\rho^2 d\Omega_3^2+dw_5^2+dw_6^2),$$

$$w_5 = 0$$

Quark injection

- Since we want homogeneous chemical potential, we turn on the component only F_{tz}
- In AdS/CFT, the chemical potential is given by,

$$\mu = A_t(r = \infty) - A_t(r = 0)$$

- This simply measures # of electric charges located at the origin

Quark injection

- To change baryon number, we modify boundary condition by adding source term,

$$\delta S = \mu_7 V_3 \mathrm{Vol}(S^3) \int dt dz \left(A_t j^t + A_z j^z \right)$$

- This describes end-points of a fundamental string (electric charges) thrown in from the outside of the system, i.e., from the boundary into the bulk.

Quark injection

- Since the end point of F-string propagates with light speed toward the center of the bulk from boundary along z-direction, and since
$$G_{tt} = -G_{zz}$$
- It propagate along the null vector
$$(v_t, v_z) = (1, -1)$$
- Therefore the source current is an arbitrary function of the variable `$t - z$'

Quark injection

- With a current conservation relation, we obtain that $j^t = j^z$, and we take the arbitrary source function as $j^t = j^z = g'(t - z)$,

$$j^t = j^z = g'(t - z)$$

- This is our input function (how we inject quarks into the bulk)
- $g \propto n_B (=\text{ baryon density})$

Quark injection

- Given this, we can determine how the flux F_{tz} on the flavor brane behaves, namely we solve

$$S_{D7inAdS} = -\mu_7 V_3 \mathrm{Vol}(S^3) \int dt dz \frac{R^8}{z^5} \sqrt{1 - \frac{z^4}{R^4} (2\pi\alpha')^2 F_{tz}^2} \, ,$$

- With input source functions

$$j^t = j^z = g'(t - z)$$

$$\delta S = \mu_7 V_3 \mathrm{Vol}(S^3) \int dt dz \left(A_t j^t + A_z j^z \right) \, ,$$

Quark injection

- We obtain solution F_{tz} ;

$$(2\pi\alpha')F_{tz} = \frac{R^2 z\, g(t-z)}{\sqrt{(2\pi\alpha')^2 R^{12} + z^6 \left(g(t-z)\right)^2}} .$$

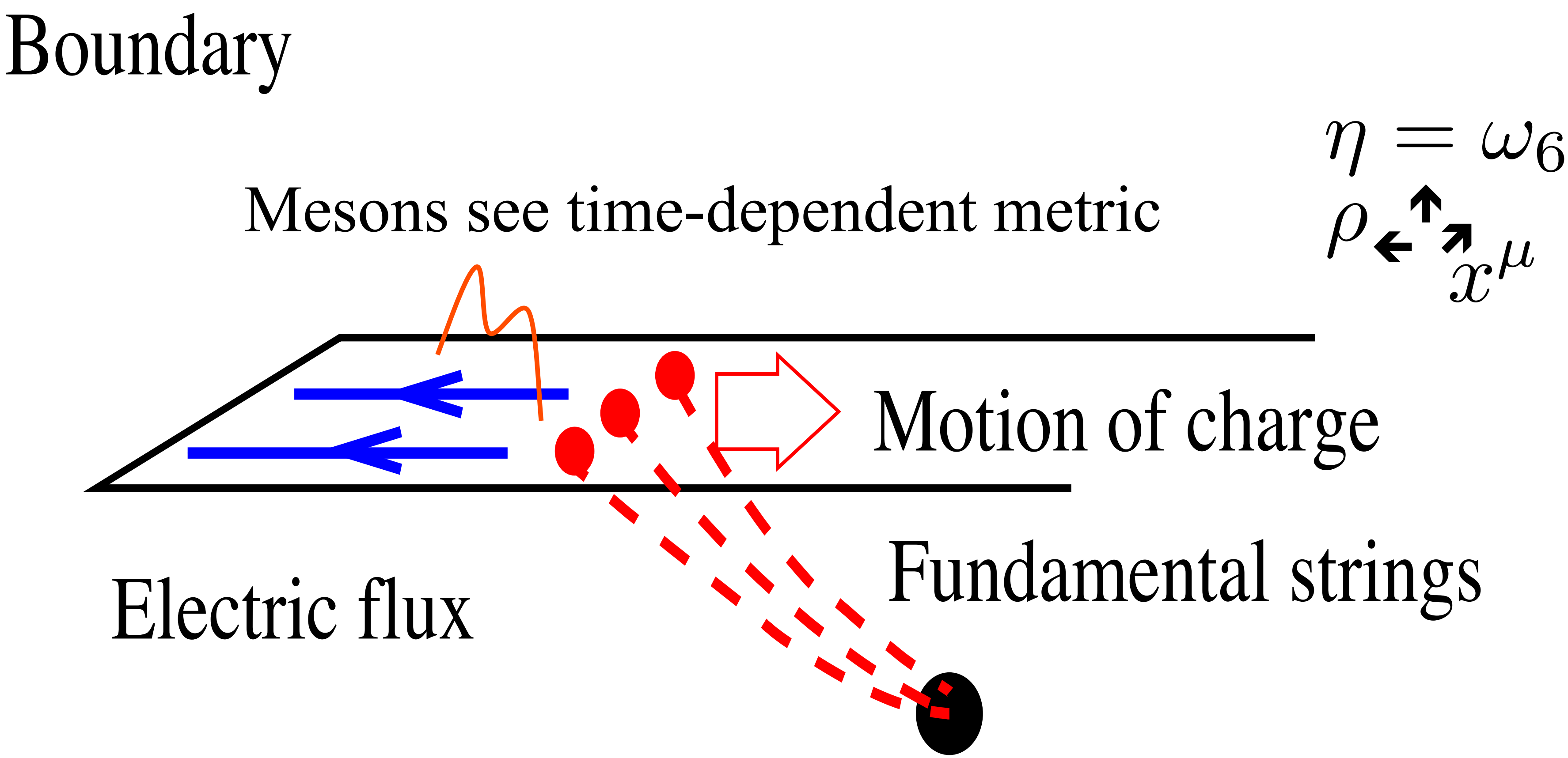
- Comparison this with DBI solution with quark source gives

$$g_{max} = (2/\pi)(2\pi\alpha')^4 \lambda\, n_{\mathrm{B\,max}}$$

Quark injection

- Given this time-dependent background, we consider small fluctuations (=mesons $\eta = \omega_6$) on the D7 branes

$$S_{D7,flux,AdS} = -\mu_7 \int d^8\xi \sqrt{-\det\left(G_{ab} + 2\pi\alpha' F_{ab}\right)},$$



$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2),$$

⇧ ⇧ ⇧

D7-brane wrapping directions

$w_5 = 0$

Quark injection

- By expanding the D7-brane action for small fluctuation $\delta\eta$ in the background solution F_{tz} , we can obtain effective metric;

$$S_{D7,flux,AdS} = -\int dtdzd^3x^i d^3\theta^I \; \frac{\sqrt{-\tilde{g}}}{2} \tilde{g}^{MN} \partial_M \delta\eta \partial_N \delta\eta + \mathcal{O}(\delta\eta^3),$$

$$-\tilde{g}_{tt} = \tilde{g}_{zz} = \mu_7^{1/3} R^{4/3} z^{-4/3} (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2)^{5/6},$$

$$\tilde{g}_{ij} = \mu_7^{1/3} R^{4/3} z^{-4/3} (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2)^{-1/6} \delta_{ij},$$

$$\tilde{g}_{IJ} = \mu_7^{1/3} R^{4/3} z^{2/3} (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2)^{-1/6} G_{IJ},$$

3-sphere directions ($\tilde{g}_{IJ}$)

Unit 3-sphere (G_{IJ})

Apparent horizon for mesons

- Given this effective metric, we can determine the apparent horizon, which is defined locally as a surface whose area variation vanishes along the null rays which is normal to the surface. The surface area at an arbitrary point in given (t, z) is

$$V_{\text{surface}} = \int d^3x^i d^3\theta^I \sqrt{(\Pi_{i=1,2,3}\ \tilde{g}_{ii})(\Pi_{I=1,2,3}\ \tilde{g}_{II})}$$

$$= V_3 \text{Vol}(S^3) \mu_7 R^4 z^{-1} (1 - z^4 R^{-4} (2\pi\alpha'^2) F_{tz}^2)^{-1/2}.$$

Apparent horizon for mesons

- The $(t\,,\,z)$ spacetime has a trivial null vector normal to the surface $(v_t,v_z) = (1,-1)$, since $g_{zz} = -g_{tt}$, so the constancy of the surface area variation along this null ray is

$$dV_{\text{surface}}\big|_{dt=-dz} = 0$$

which yields

Eq. for apparent horizon

$$(2\pi\alpha')^2 R^{12} - 2z^6 g^2 + 2z^7 g g' = 0$$

where $j^t = j^z = g'(t-z)$

Thermalization time-scale

- We can estimate thermalization time-scale by dimensional analysis; for `*n, k'*= *non-negative.*
- Setting $\xi = t - z$, for `*n, k'*= *non-negative*
- We imagine that baryon chemical potential increase due to quark injection, reach maximam in time-scle ~ $1/w$ by power n

$$g(\xi) \sim g_{\max}(w\xi)^n \,,\; g'(\xi) \sim w g_{\max}(w\xi)^{n-1} \,.$$

Thermalization time-scale

- Then if there is a solution to previous eq. for apparent horizon it scales as

$$t_{\rm th} \sim \left(\frac{(2\pi\alpha')^2 R^{12}}{g_{\rm max}^2 w^{2n}} \right)^{1/(6+2n)} \sim \left(\frac{\lambda}{n_{\rm B}^2 w^{2n}} \right)^{1/(6+2n)}$$

- Generically;

$$t_{\rm th} \sim \left(\frac{\lambda}{n_{\rm B}^2 w^k} \right)^{1/(6+k)}$$

Heavy ion collision example

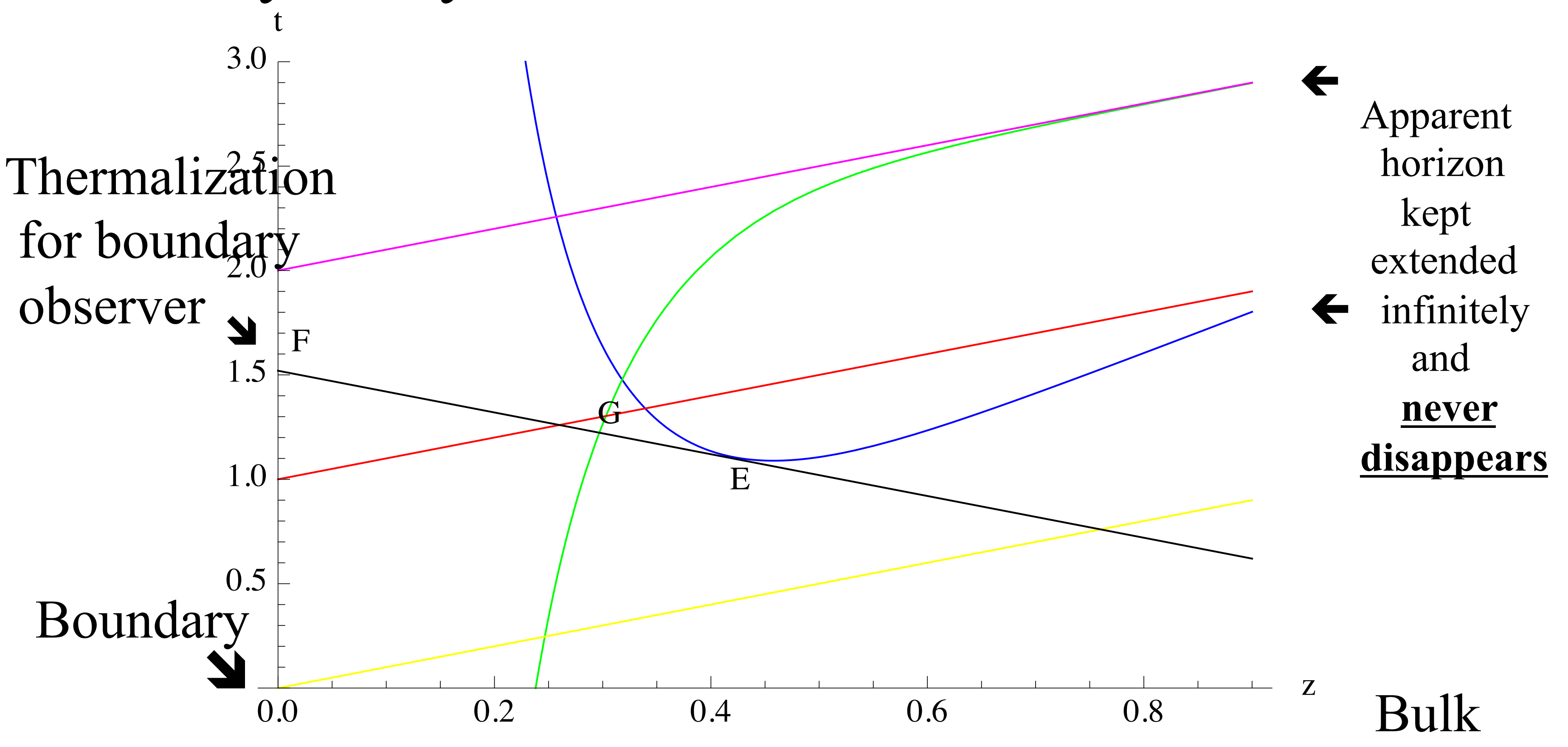
- Let us choose our input source function such that two heavy ion collision

$$g(t-z) = \begin{cases} 0 & (\xi < 0) \\ g_{\max}\, w\,\xi & (0 < \xi < 1/w) \\ g_{\max}\,(2-\xi w) & (1/w < \xi < 2/w) \\ 0 & (2/w < \xi) \end{cases}$$

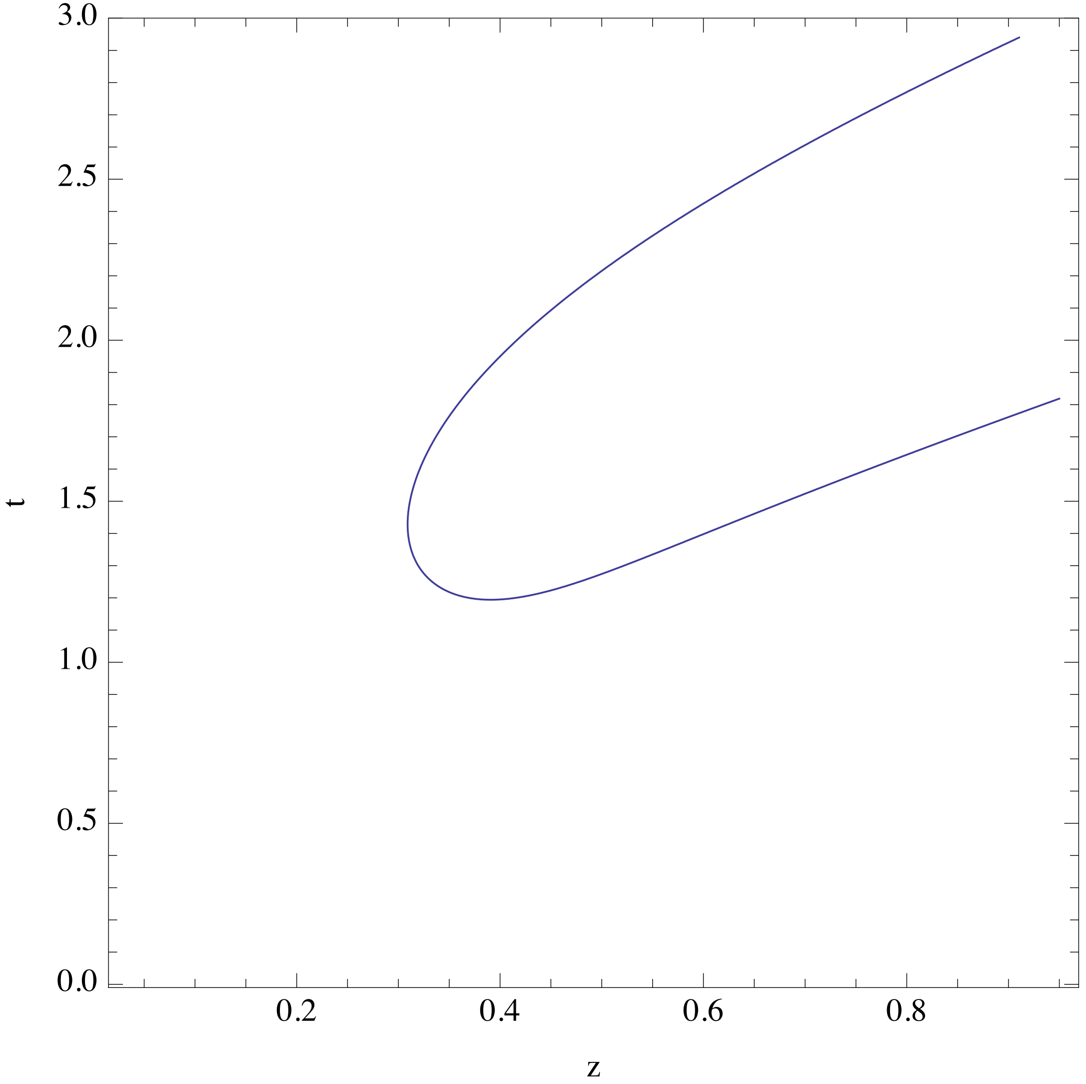
- This is the situation where chemical potential increase linearly and reach max and decrease,

Heavy ion collision example

- Formation of apparent horizon is determined analytically in the unit $(\lambda/(2\pi n_{\rm B}^2))^{1/6} = 1/w$



Heavy ion collision example



Heavy ion collision example

- Dependent on the parameters, careful analysis shows that the time-scale for thermalization is deterimed as

$$t_{\rm th} = \min_{k=0,1,2} \left\{ \mathcal{O}\left(\frac{\lambda}{n_{\rm B}^2 w^k}\right)^{1/(6+k)} \right\}$$

- Consistent with demensional analysis
- λ dependence is very weak (since $k >= 0$)

Comparison with Data

- In heavy ion collision,

Typical nuclear density ~ 0.2 [fm]$^{-3}$

$$n_{\rm B} \sim 2\gamma n_{\rm N}$$

Lorentz factor ~ E/m ~ 100 (RHIC), ~2000 (LHC)

$$1/w \sim 2A^{1/3}\gamma\,[{\rm fm/c}]$$

A is nucleon number
For Au, A = 197

$$\lambda \sim \mathcal{O}(10) < 100$$

Comparison with Data

$$t_{\rm th} = \min_{k=0,1,2}\left\{\mathcal{O}\left(\frac{\lambda}{n_{\rm B}^2 w^k}\right)^{1/(6+k)}\right\}$$

$$\left(\frac{\lambda}{w^2 n_{\rm B}^2}\right)^{1/8} \sim \left(\frac{A^{2/3}\lambda}{\gamma^4 n_{\rm N}^2}\right)^{1/8} \sim 0.24 \times \lambda^{1/8}\,[\mathrm{fm/c}]\,.$$

$$\left(\frac{\lambda}{w n_{\rm B}^2}\right)^{1/7} \sim \left(\frac{A^{1/3}\lambda}{\gamma^3 n_{\rm N}^2}\right)^{1/7} \sim 0.30 \times \lambda^{1/7}\,[\mathrm{fm/c}]\,,$$

$$\left(\frac{\lambda}{n_{\rm B}^2}\right)^{1/6} \sim \left(\frac{\lambda}{\gamma^2 n_{\rm N}^2}\right)^{1/6} \sim 0.39 \times \lambda^{1/6}\,[\mathrm{fm/c}]\,.$$

Comparison with Data

- All shows that time-scale is for

$$\lambda \sim \mathcal{O}(10) < 100$$

$$t_{\rm th} < 1\,[\mathrm{fm/c}]$$

- In LHC, due to Lorentz factor difference, thermalization is farther smaller as

$$t_{\rm th} \lesssim \mathcal{O}\left(0.1\right)\ [\mathrm{fm/c}]\,.$$

<u>Very rapid thermalization at LHC!?</u>

Universality

- We have shown the scalar meson thermalization in the massless $\mathcal{N}$=2 SUSY conformal theory
- It is straightforward to extend the calculation of the thermalization for other degrees of freedom like vector mesons
- The results are almost the same, does not change the results ➔ some universality?

Universality

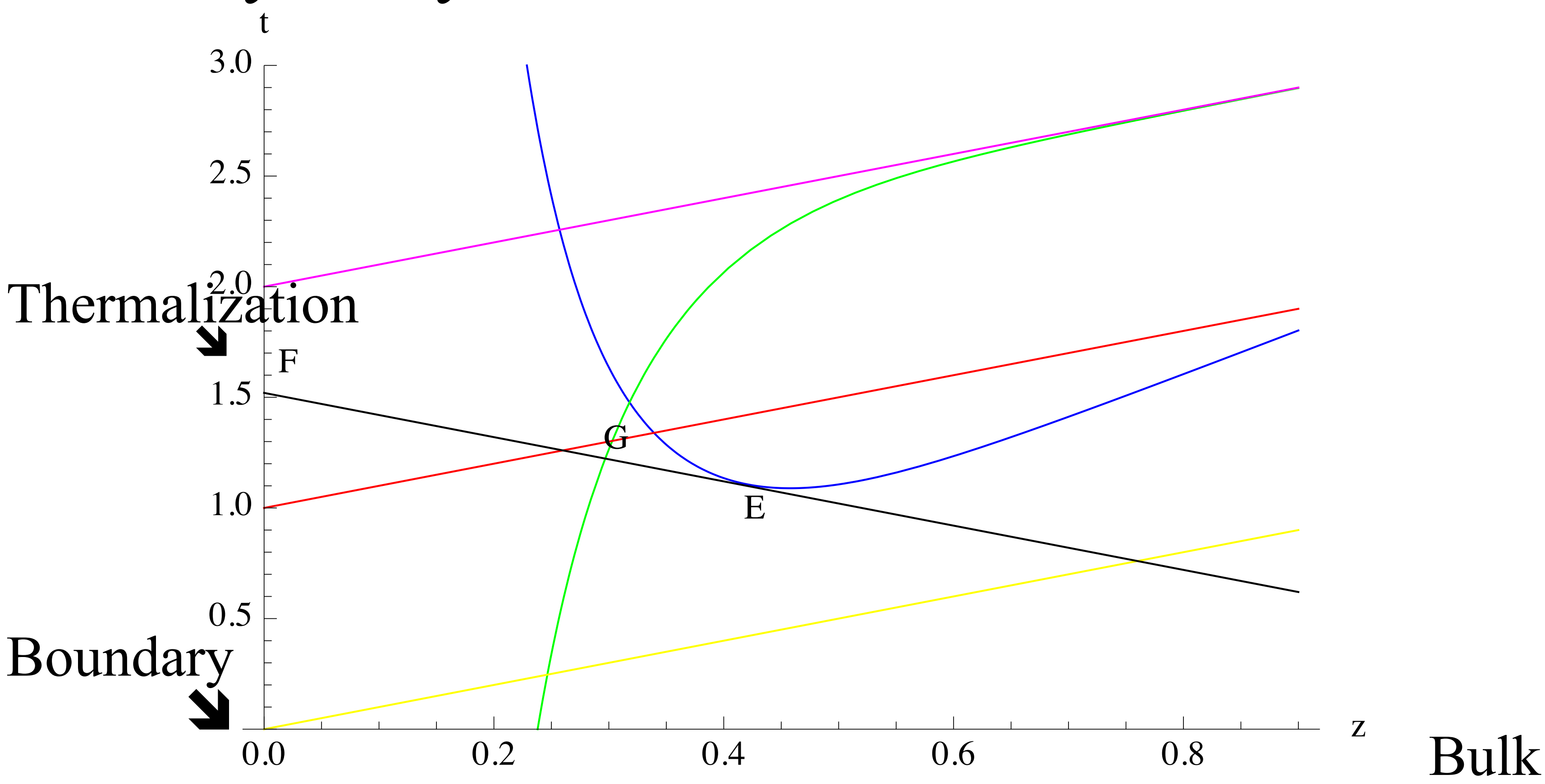
- The key points is that thermalization occurs near the boundary at the regime where

$$z < z_c$$

- Therefore *even if we modify the IR regime into non-conformal theory, we expect that our estimation is not influenced much*
- Our setup works for non-conformal theory where IR is modified from AdS_5

Heavy ion collision example

- Formation of apparent horizon is determined analytically in the unit $(\lambda/(2\pi n_{\mathrm{B}}^2))^{1/6} = 1/w$



Universality

- Our estimation for RHIC input case gives the thermalization time scale as

$$c\,t_{th} \sim z_c \lesssim 1[\mathrm{fm}]$$

- This corresponds to the energy scale as

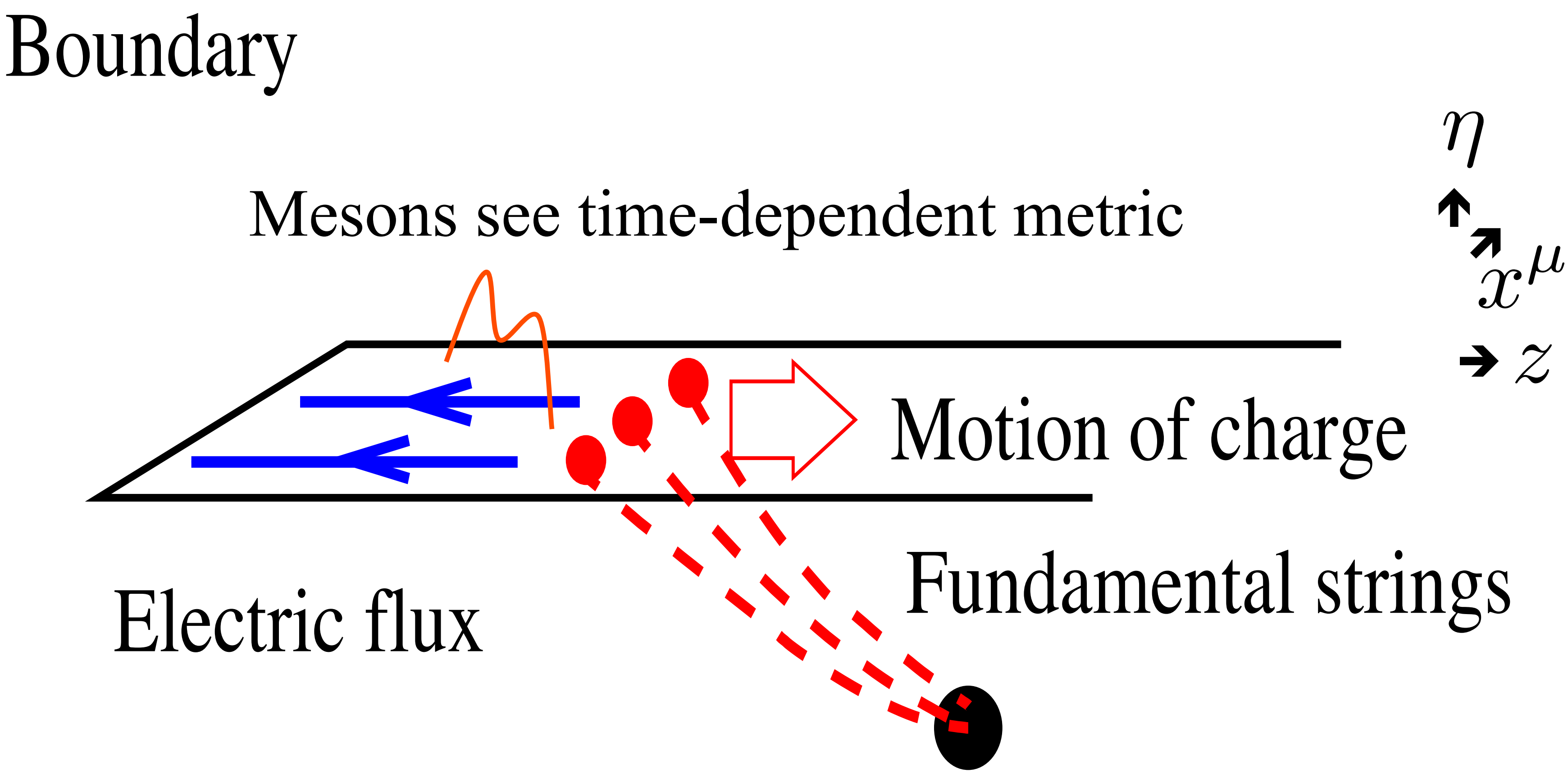
$$E_c \gtrsim 200[\mathrm{MeV}]$$

- As far as we deform bulk to non-conformal below this scale, we expect the same order thermalization timescale

Real world QCD Λ_{QCD} ~ 200 Mev is validity bound!

Universality

- We can also consider quark mass effects, which breaks UV conformality
- Noticing that in the effective metric always appear as $R^2/z^2 + \eta^2/R^2$
- By replacing $R^2/z^2 \to R^2/z^2 + \eta^2/R^2$ we obtain the quark mass case



Boundary
Mesons see time-dependent metric
η
x^μ
z
Motion of charge
Electric flux
Fundamental strings

Universality

- However we used bulk geometry only

$$c\,t_{th} \sim z_c \lesssim 1[\mathrm{fm}]$$

- Therefore if

$$R^2/z_c^2 \gg \eta^2/R^2$$

  which is equivalent to

$$m_q \ll (\sqrt{2}\lambda n_{\mathrm{B}}/\pi)^{1/3} \sim 100\,\lambda^{1/3}\ [\mathrm{MeV}]$$

- Our argument is not influenced by quark mass for flavor colors (up, down, strange)

Summary

- Thermalization for meson sectors are much more under control than glueball sectors
- Approximates the chemical potential change as if situations for the RHIC and LHC
- Found very rapid themalization time-scale for < 1 [fm/c], faster for LHC < 0.1 [fm/c]
- Universality; real world QCD is validity bound
- Non-conformality at IR won’t spoil the results
- Small quark mass won’t spoil the results either